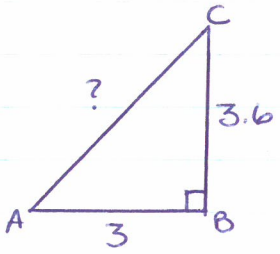
Investment 1 p156

1) Use the Pythagorean Theorem to calculate the missing measure in each right triangle.

a)



$a^2 + b^2 = c^2$

$(3.6)^2 + (3)^2 = c^2$

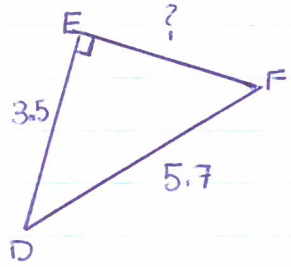
$12.96 + 9 = c^2$

$21.96 = c^2$

$\sqrt{21.96} = c$

$4.69 \approx c$ $\quad m\overline{AC} \approx 4.69$

b)



$a^2 + b^2 = c^2$

$a^2 + (3.5)^2 = (5.7)^2$

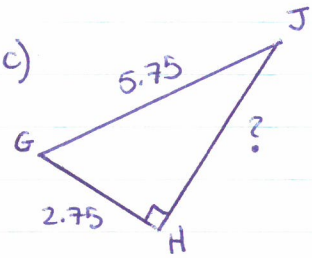
$a^2 + 12.25 = 32.49$

$a^2 = 32.49 - 12.25$

$a^2 = 20.24$

$a \approx 4.499$ $\quad m\overline{EF} \approx 4.50$

c)



$a^2 + b^2 = c^2$

$(2.75)^2 + b^2 = (5.75)^2$

$7.5625 + b^2 = 30.0625$

$b^2 = 30.0625 - 7.5625$

$b^2 = 25.5$

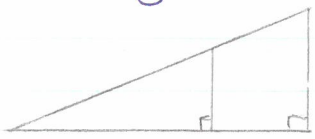
$\sqrt{b^2} = \sqrt{25.5}$

$b \approx 5.0498$ $\quad m\overline{JH} \approx 5.05$

2) TRUE or FALSE

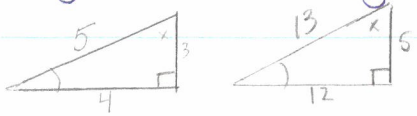
a) All right triangles that contain a 20° angle are similar.

TRUE



b) All right triangles with a leg measuring 5cm are similar.

FALSE



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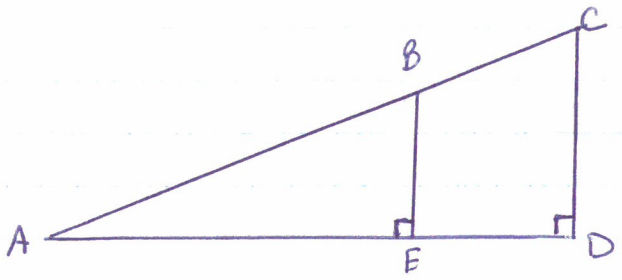
3) Two similar right triangles are shown. Determine whether the given ratios are proportional.



a) $\frac{m\overline{CD}}{m\overline{BE}}$ and $\frac{m\overline{AC}}{m\overline{AB}}$ Yes

b) $\frac{m\overline{CD}}{m\overline{AC}}$ and $\frac{m\overline{BE}}{m\overline{AB}}$ Yes

4a) What must be done to the following proportions to produce an equality between two sines of the same angle?

$$\frac{m\overline{CD}}{m\overline{AC}} = \frac{m\overline{AC}}{m\overline{AB}}$$ Interchange the means

b) What must be done to the following proportions to produce an equality between two cosines of the same angle?

$$\frac{m\overline{AD}}{m\overline{AE}} = \frac{m\overline{AC}}{m\overline{AB}}$$ Interchange the means

c) What must be done to the following proportions to produce an equality between two tangents of the same angle?

$$\frac{m\overline{CD}}{m\overline{BE}} = \frac{m\overline{AD}}{m\overline{AE}}$$ Interchange the means

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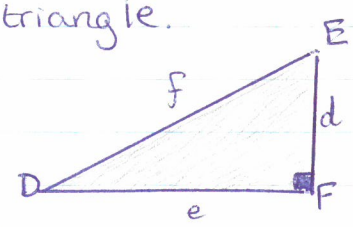
5) Here are six ratios in a right triangle.

$$\frac{d}{e}, \frac{e}{f}, \frac{f}{d}, \frac{e}{d}, \frac{f}{e}, \frac{d}{f}$$



Find the one that represents

a) $\sin(D) = \frac{d}{f}$

b) $\cos(D) = \frac{e}{f}$

c) $\tan(E) = \frac{e}{d}$

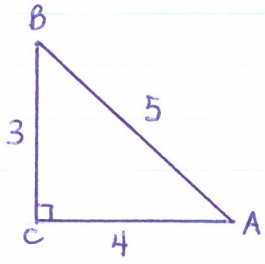
d) $\cos(E) = \frac{d}{f}$

e) $\sin(E) = \frac{e}{f}$

f) $\tan(D) = \frac{d}{e}$

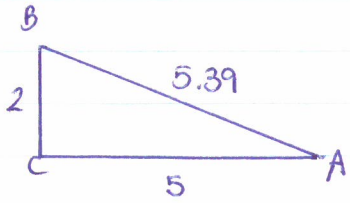
6) Find $\sin(A)$ in each of the following right triangles.

a)



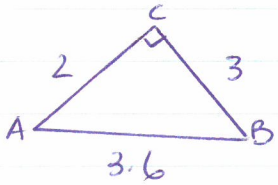
$$\sin(A) = \frac{3}{5} = 0.6$$

b)



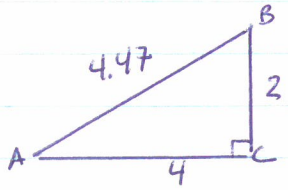
$$\sin(A) = \frac{2}{5.39} = 0.3711$$

c)



$$\sin(A) = \frac{3}{3.6} = 0.8333$$

d)



$$\sin(A) = \frac{2}{4.47} = 0.4474$$

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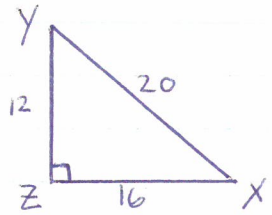
7) For each right triangle, find the value of the sine, cosine and tangent of angle X

a)

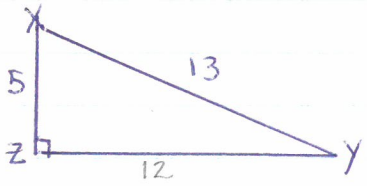


$\sin(x) = \frac{12}{20} \approx 0.6$

$\cos(x) = \frac{16}{20} \approx 0.8$

$\tan(x) = \frac{12}{16} \approx 0.75$

b)

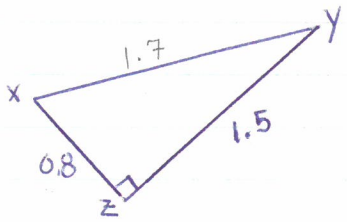


$\sin(x) = \frac{12}{13} \approx 0.9231$

$\cos(x) = \frac{5}{13} \approx 0.3846$

$\tan(x) = \frac{12}{5} \approx 2.4$

c)

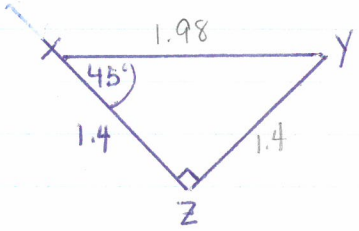


$\sin(x) = \frac{1.5}{1.7} \approx 0.8824$

$\cos(x) = \frac{0.8}{1.7} = 0.4706$

$\tan(x) = \frac{1.5}{0.8} = 1.875$

d)



$\sin(x) = \frac{1.4}{1.98} \approx 0.7071$

$\cos(x) = \frac{1.4}{1.98} \approx 0.7071$

$\tan(x) = \frac{1.4}{1.4} = 1$

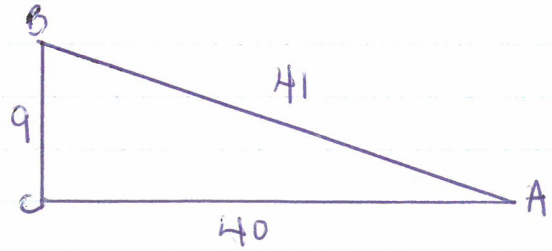
8) Give the possible names of each ratio

a) $\frac{9}{40} = \tan(A)$

b) $\frac{40}{41} = \sin(B)$ OR $\cos(A)$

c) $\frac{9}{41} = \sin(A)$ OR $\cos(B)$

d) $\frac{40}{9} = \tan(B)$



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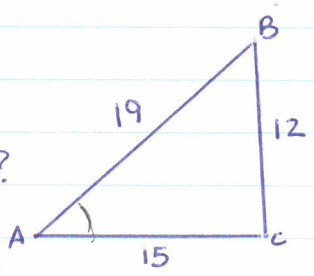
#9 $m\angle A$	#9a) sin A	#9b) cos A	#9c) tan A
0°	0	1	0
10°	0.1736	0.9848	0.1763
20°	0.3420	0.9397	0.3640
30°	0.5	0.8660	0.5774
40°	0.6428	0.7660	0.8391
50°	0.7660	0.6428	1.1918
60°	0.8660	0.5	1.7321
70°	0.9397	0.3420	2.7475
80°	0.9848	0.1736	5.6713
90°	1	0	—

10) Consider the triangle shown on the right.



a) Which side is opposite angle A?

$\overline{BC}$

b) Which side is adjacent to angle A?

would say $\overline{AC}$

Can't say: $\triangle ABC$ is not a right triangle

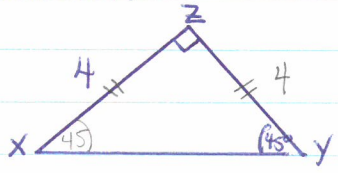
c) Can you calculate sin(A)? Why?

No. $\triangle ABC$ is not a right triangle

$(12^2 + 15^2 \neq 19^2)$

≈ 19.20

11) Is it possible to calculate tan 45° using the given right triangle? Justify your answer.



A right triangle with a 45° angle is an isosceles triangle

$\angle X = \angle Y = 45°$

$mXZ = mXY = 4$

$\tan 45° = \frac{4}{4} = 1$

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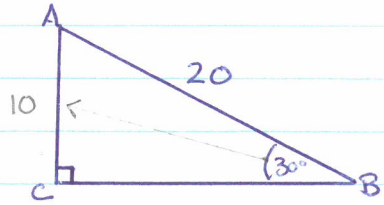
12) In a right triangle, a side measuring one-half the hypotenuse is opposite a 30° angle. Find the missing measure and calculate the cos

Hypotenuse = 20

$\therefore m\overline{AC} = 10$



$\cos(A) = \frac{A}{H}$ $\quad \cos(60) = \frac{10}{20} = 0.5$

13) Using the information indicated on the figure find:

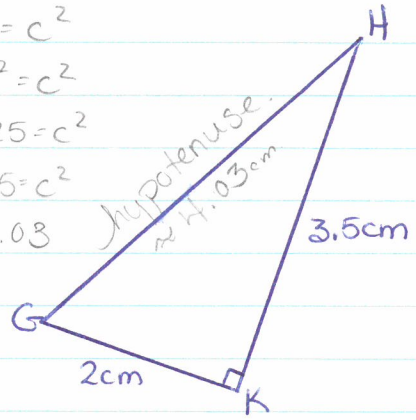
$a^2 + b^2 = c^2$

$2^2 + 3.5^2 = c^2$

$4 + 12.25 = c^2$

$16.25 = c^2$

$c \approx 4.03$



a) $m\overline{GH} \approx 4.03\text{cm}$

b) $\tan G = \frac{3.5}{2} = 1.75$

c) $\tan H = \frac{2}{3.5} = 0.5714$

d) $\sin G = \frac{3.5}{4.03} \approx 0.8685$

e) $\cos H = \frac{3.5}{4.03} \approx 0.8685$

f) $\sin H = \frac{2}{4.03} \approx 0.4963$

g) $\cos G = \frac{2}{4.03} \approx 0.4963$

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14) Angles A and B are the two acute angles of a right triangle. Determine which of the two is greater if

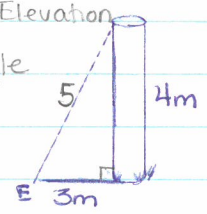
a) $\sin(A) > \sin(B)$ Ans: $\angle A$

b) $\tan(A) < \tan(B)$ Ans: $\angle B$

c) $\cos(A) > \cos(B)$ Ans: $\angle B$

15) At a given time of day, a 4m pole casts a 3m shadow. Express these two measures in terms of the sun's angle of elevation.

Angle of Elevation is an angle above the horizontal



$4 = 5 \sin E$

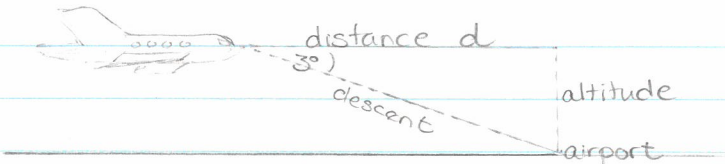
$3 = 5 \cos E$

$\frac{\sin(E)}{1} = \frac{4}{5}$

cross-multiply

$5 \sin E = 4$

16) A plane is at a horizontal distance "d" from an airport. To land at the airport, it must begin its descent at an angle of depression of 3°. Express the altitude of the plane in terms of d



$\tan(3°) = \frac{\text{altitude}}{\text{distance}}$

$\text{distance}(\tan(3)) = \text{altitude}$

$\text{Altitude} = d \tan(3°)$

17) Two ladders of different lengths are placed against a wall. The base of the ladders are at the same distance from the wall. Which of the ladders forms a triangle in which:

a) $\sin C$ is greater?

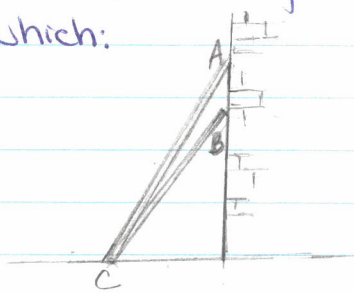
Ans: Ladder AC

ex (AC) $\sin C = \frac{4}{12} = \frac{1}{3}$

(BC) $\sin C = \frac{3}{12} = \frac{1}{4}$

b) $\cos C$ is smaller?

Ans: Ladder AC



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18) Nadia builds a doghouse whose roof has the same shape as an isosceles right triangle. The length of leg $\overline{BC}$ is 1m.

a) What is the measurement of $\overline{AC}$?

$m\overline{AC} = 1m$

b) What is the length of the hypotenuse?

Pythagoras: $a^2 + b^2 = c^2$

$(m\overline{AC})^2 + (m\overline{BC})^2 = (m\overline{AB})^2$

$1^2 + 1^2 = (m\overline{AB})^2$

$1 + 1 = (m\overline{AB})^2$

$2 = (m\overline{AB})^2$

$\sqrt{2} = m\overline{AB}$ → hypotenuse.

Ans: $m\overline{AB} = \sqrt{2} \approx 1.41$

c) Find the value of each trigonometric ratio for angle A.

$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \approx 0.7071$

$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \approx 0.7071$

$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$

d) What are the values of these ratios if the shape of the roof is identical, but $m\overline{BC} = 1.2m$

If $m\overline{AC} = m\overline{BC} = 1.2$

Then $m\overline{AB} = \sqrt{2.88}$

$\sin(A) = \frac{1.2}{\sqrt{2.88}} \approx 0.7071$

$\cos(A) = \frac{1.2}{\sqrt{2.88}} \approx 0.7071$

$\tan(A) = \frac{1.2}{1.2} = 1$

The ratios are the same

18 e) What is the measure of angle A?

$m\angle A = 45°$

f) Using a calculator, express the following in decimal form.

1) $\sin 45° =$
2) $\cos 45° =$
3) $\frac{1}{\sqrt{2}} =$
4) $\frac{\sqrt{2}}{2} =$
5) $\tan 45° =$

19) A right triangle has a 30° angle. Its hypotenuse measures 2 units.

a) Find the length of the other two sides. Justify step of your method.

The leg opposite a 30° angle of a right angle triangle measures half the length of the hypotenuse.

∴ The length of the leg opposite to the 30° angle measures 1 unit.

If you know 2 sides of a right triangle you can solve for the third side using Pythagoras.

$a^2 + b^2 = c^2$

$1^2 + b^2 = 2^2$

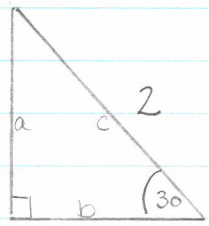
$1 + b^2 = 4$

$b^2 = 4 - 1$

$b^2 = 3$

$b = \sqrt{3}$

$b \approx 1.73$ units



19 b) Find the three trigonometric ratios for the 30° angle and the other acute angle.

$\sin(30) = \frac{1}{2} = 0.5$ $\quad \cos(30) = \frac{\sqrt{3}}{2} \approx 0.8660$ $\quad \tan(30) = \frac{1}{\sqrt{3}} \approx 0.5774$

$\sin(60) = \frac{\sqrt{3}}{2} \approx 0.8660$ $\quad \cos(60) = \frac{1}{2} = 0.5$ $\quad \tan(60) = \frac{\sqrt{3}}{2} \approx 1.7321$

c) Use a calculator to find the decimal form of the following:

1) $\cos(30) = 0.8660$
2) $\sin(60) = 0.8660$
3) $\frac{\sqrt{3}}{2} = 0.8660$
4) $\tan(30) = 0.5774$
5) $\frac{1}{\sqrt{3}} = 0.5774$
6) $\frac{\sqrt{3}}{3} = 0.5774$