SOLUTIONS:

Review Questions: Change in x, Change in y, Distance, Midpoint, Division Point

1a) $E(-7,3)$ and $F(-4,-1)$

Point E to *Point F*	*Point F* to *Point E*	Distance	Midpoint
$\Delta x = x_2 - x_1$ $= (-4) - (-7)$ $= -4 + 7$ $= 3$ $\Delta y = y_2 - y_1$ $= (-1) - (3)$ $= -1 - 3$ $= -4$	$\Delta x = x_2 - x_1$ $= (-7) - (-4)$ $= -7 + 4$ $= -3$ $\Delta y = y_2 - y_1$ $= (3) - (-1)$ $= 3 + 1$ $= 4$	$d(A,B) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $= \sqrt{(3)^2 + (-4)^2}$ $= \sqrt{9 + 16}$ $= \sqrt{25}$ $= 5$ units	$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$ $\left(\frac{(-7) + (-4)}{2}, \frac{(3) + (-1)}{2}\right)$ $\left(\frac{-11}{2}, \frac{2}{2}\right)$ $(5.5,\ 1)$ *Midpoint* $(5.5, 1)$

1b) $E(-8,5)$ and $F(36,-28)$

Point E to *Point F*	*Point F* to *Point E*	Distance	Midpoint
$\Delta x = x_2 - x_1$ $= (36) - (-8)$ $= 36 + 8$ $= 44$ $\Delta y = y_2 - y_1$ $= (-28) - (5)$ $= -28 - 5$ $= -33$	$\Delta x = x_2 - x_1$ $= (-8) - (36)$ $= -8 - 36$ $= -44$ $\Delta y = y_2 - y_1$ $= (5) - (-28)$ $= 5 + 28$ $= 33$	$d(A,B) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $= \sqrt{(44)^2 + (-33)^2}$ $= \sqrt{1936 + 1089}$ $= \sqrt{3025}$ $= 55$ units	$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$ $\left(\frac{(-8) + (36)}{2}, \frac{(5) + (-28)}{2}\right)$ $\left(\frac{28}{2}, \frac{-23}{2}\right)$ $(14,\ -11.5)$ *Midpoint* $(14,\ -11.5)$

1c) $E(-73,14)$ and $F(35,9)$

Point E to *Point F*	*Point F* to *Point E*	Distance	Midpoint
$\Delta x = x_2 - x_1$ $= (35) - (-73)$ $= 35 + 73$ $= 108$ $\Delta y = y_2 - y_1$ $= (9) - (14)$ $= 9 - 14$ $= -5$	$\Delta x = x_2 - x_1$ $= (-73) - (35)$ $= -73 - 35$ $= -108$ $\Delta y = y_2 - y_1$ $= (14) - (9)$ $= 14 - 9$ $= 5$	$d(A,B) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $= \sqrt{(108)^2 + (-5)^2}$ $= \sqrt{1164 + 25}$ $= \sqrt{1189}$ ≈ 108.12 units	$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$ $\left(\frac{(-73) + (35)}{2}, \frac{(14) + (9)}{2}\right)$ $\left(\frac{-38}{2}, \frac{23}{2}\right)$ $(-19,\ 11.5)$ *Midpoint* $(-19,\ 11.5)$

1d)	$E(-1,4)$ and $F(8,-8)$			
	Point E to *Point F*	*Point F* to *Point E*	Distance	Midpoint
	$\Delta x = x_2 - x_1$ $= (8) - (-1)$ $= 8 + 1$ $= 9$ $\Delta y = y_2 - y_1$ $= (-8) - (4)$ $= -8 - 4$ $= -12$	$\Delta x = x_2 - x_1$ $= (-1) - (8)$ $= -1 - 8$ $= -9$ $\Delta y = y_2 - y_1$ $= (4) - (-8)$ $= 4 + 8$ $= 12$	$d(A,B) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $= \sqrt{(9)^2 + (-12)^2}$ $= \sqrt{81 + 144}$ $= \sqrt{225}$ $= 15$ units	$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$ $\left(\frac{(-1)+(8)}{2}, \frac{(4)+(-8)}{2}\right)$ $\left(\frac{7}{2}, \frac{-4}{2}\right)$ $(3.5, -2)$ *Midpoint* $(3.5, -2)$

1e)	$E(3,17)$ and $F(15,24.5)$			
	Point E to *Point F*	*Point F* to *Point E*	Distance	Midpoint
	$\Delta x = x_2 - x_1$ $= (15) - (3)$ $= 15 - 3$ $= 12$ $\Delta y = y_2 - y_1$ $= (24.5) - (17)$ $= 24.5 - 17$ $= 7.5$	$\Delta x = x_2 - x_1$ $= (3) - (15)$ $= 3 - 15$ $= -12$ $\Delta y = y_2 - y_1$ $= (17) - (24.5)$ $= 17 - 24.5$ $= -7.5$	$d(A,B) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $= \sqrt{(12)^2 + (-7.5)^2}$ $= \sqrt{144 + 56.25}$ $= \sqrt{200.25}$ $= 14.15$ units	$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$ $\left(\frac{(3)+(15)}{2}, \frac{(17)+(24.5)}{2}\right)$ $\left(\frac{18}{2}, \frac{41.5}{2}\right)$ $(9, 20.75)$ *Midpoint* $(9, 20.75)$

2) From *Point E* $(-7, 1)$ to *Point F* $(8,-34)$ ratio 1:4

Workshop $A(1,-4)$ $B(6,4)$

$a = 1$
$b = 4$
$a + b = 5$

$x_1 = -7$
$y_1 = 1$

$\Delta x = x_2 - x_1$
$= (8) - (-7)$
$= 8 + 7$
$= 15$

$\Delta y = y_2 - y_1$
$= (-34) - (1)$
$= -34 - 1$
$= -35$

$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x)$
$= (-7) + \left(\frac{1}{5}\right)(15)$
$= (-7) + \left(\frac{1}{5}\right)(15)$
$= (-7) + \left(\frac{15}{5}\right)$
$= (-7) + (3)$
$= -4$

$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y)$
$= (1) + \left(\frac{1}{5}\right)(-35)$
$= (1) + \left(\frac{1}{5}\right)(-35)$
$= (1) + \left(\frac{-35}{5}\right)$
$= (1) + (-7)$
$= -6$

Point $(-4,-6)$

3) *Point* $E(2,3)$ to *Point* $F(-26,7)$ fraction $\frac{3}{4}$

Workshop $A(1,-4)$ $B(6,4)$

$a = 3$
$b = 1$
$a + b = 4$

$x_1 = 2$
$y_1 = 3$

$\Delta x = x_2 - x_1$
$= (-26) - (2)$
$= -26 - 2$
$= -28$

$\Delta y = y_2 - y_1$
$= (7) - (3)$
$= 7 - 3$
$= 4$

$$x_D = x_1 + \left(\tfrac{a}{a+b}\right)(\Delta x)$$
$$= (2) + \left(\tfrac{3}{4}\right)(-28)$$
$$= (2) + \left(\tfrac{3}{4}\right)(-28)$$
$$= (2) + \left(\tfrac{-84}{4}\right)$$
$$= (2) + (-21)$$
$$= -19$$

$$y_D = y_1 + \left(\tfrac{a}{a+b}\right)(\Delta y)$$
$$= (3) + \left(\tfrac{3}{4}\right)(4)$$
$$= (3) + \left(\tfrac{3}{4}\right)(4)$$
$$= (3) + \left(\tfrac{12}{4}\right)$$
$$= (3) + (3)$$
$$= 6$$

Point $(-19, 6)$

4) Find the endpoint E of $\overline{EF}$ with the midpoint $M(-3,2)$ and endpoint $F(6,-14)$

$$x_m = \tfrac{x_1 + x_2}{2} \qquad y_m = \tfrac{y_1 + y_2}{2}$$

$$(-3) = \tfrac{(6) + (x)}{2} \qquad (2) = \tfrac{(-14) + (y)}{2}$$

$$(2)(-3) = \left(\tfrac{(6) + (x)}{2}\right)(2) \qquad (2)(2) = \left(\tfrac{(-14) + (y)}{2}\right)(2)$$

$$-6 = 6 + x \qquad 4 = (-14 + y)$$

$$-12 = x \qquad 18 = y$$

Midpoint $(-12, 18)$