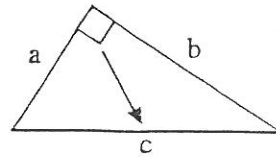
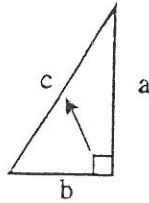
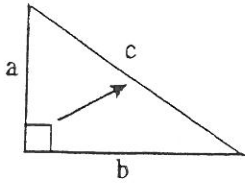


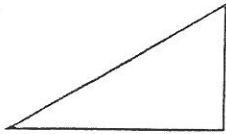
7.2 LABELLING RIGHT ANGLE TRIANGLES

All **right angle triangles** are labelled the same way. The side opposite the right angle is called the **hypotenuse** and is always labelled 'c'. The other two sides are labelled 'a' for the **altitude** and 'b' for the **base** as shown in the three examples below.

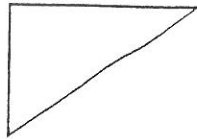


A. Label the hypotenuse of each triangle with the lower case letter 'c' and each right angle with a capital letter 'C'. (Use a protractor if necessary.)

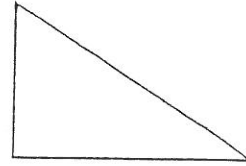
1.



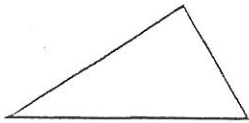
2.



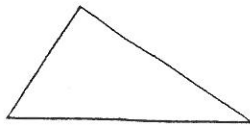
3.



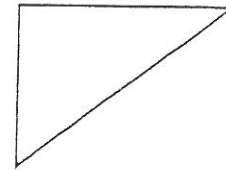
4.



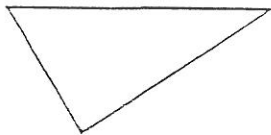
5.



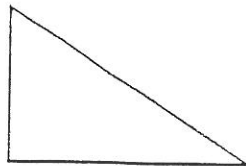
6.



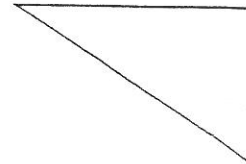
7.



8.



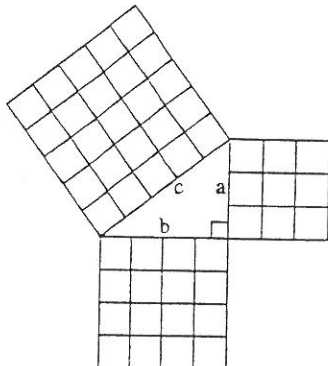
9.



7.3 AREA OF SQUARES USING THE PYTHAGOREAN THEOREM

In 552 B.C., in ancient Greece, a man called **Pythagoras** was born and before his death in 507 B.C. he was to become one of the greatest mathematicians of all time. While working with **right-angle triangles**, Pythagoras discovered what we now call the **Pythagorean Theorem**.

In any right-angle triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides. ($c^2 = a^2 + b^2$)



The diagram on the left illustrates that the area of a square drawn on the hypotenuse is equal to the sums of the areas of the squares drawn on the other two sides.

$$\begin{aligned}
 c^2 &= a^2 + b^2 \\
 5^2 &= 3^2 + 4^2 \\
 25 \text{ units}^2 &= 9 \text{ units}^2 + 16 \text{ units}^2 \\
 25 \text{ units}^2 &= 25 \text{ units}^2
 \end{aligned}$$

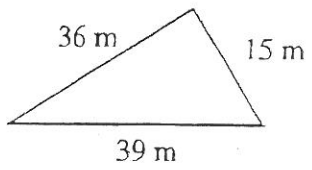
$$216 \text{ m}^2 = 17 \text{ m}^2 + 5 \text{ m}^2$$

A. Complete the table below to determine whether or not the three numbers given can be the sides of a right angle triangle, or in algebraic terms, is $c^2 = a^2 + b^2$? (The first one is already done for you.)

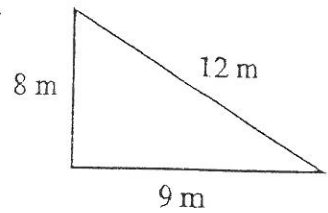
a	a ²	b	b ²	a ² + b ²	c	c ²	Is c ² = a ² + b ² ?
12	144	16	256	400	30	900	NO
5		12			13		
20		21			29		
24		70			74		
7		8			9		

B. Using the triangles below, complete the table and determine which of the triangles are right angle triangles. (The first one is already done for you.)

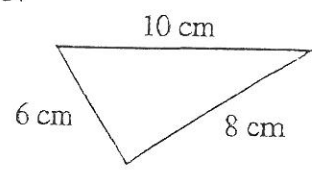
1.



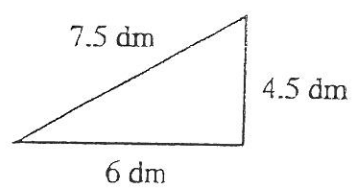
2.



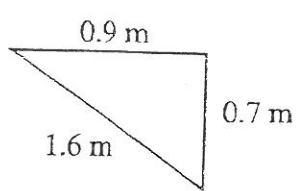
3.



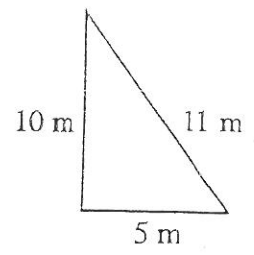
4.



5.



6.



Area of the Square Drawn on One Side	Area of the Square Drawn on Second Side	Sum of the Area of the Squares on 1st Two Sides	Area of the Square Drawn on Longest Side	Is This a Right Angle Triangle? YES or NO?
1. 225 m ²	1296 m ²	1521 m ²	1521 m ²	YES
2.				
3.				
4.				
5.				
6.				

7.4 PYTHAGOREAN TRIPLETS OR TRIADS

Certain sets of numbers are known as Pythagorean **triplets** or **triads** because they can be used to make up the sides of a right angle triangle. The set of numbers (3, 4, 5) is a triplet or a triad because by using the Theorem of Pythagoras, they satisfy the condition $c^2 = a^2 + b^2$, where the largest number is always the 'c' in our formula or the number associated with the hypotenuse.

EXAMPLE #1: Are the numbers 5, 12 and 13 a Pythagorean triplet?

$$\begin{aligned}c^2 &= a^2 + b^2 \\13^2 &= 5^2 + 12^2 \\169 &= 25 + 144 \\169 &= 169\end{aligned}$$

Yes, the numbers 5, 12 and 13 form a triplet.

EXAMPLE #2: Are the numbers 6, 8 and 10 a Pythagorean triad?

$$\begin{aligned}c^2 &= a^2 + b^2 \\10^2 &= 6^2 + 8^2 \\100 &= 36 + 64 \\100 &= 100\end{aligned}$$

Yes, the number 6, 8 and 10 form a triad.

A. State whether each of the following is either a Pythagorean triad or triplet.

1. 5, 6, 7

2. 21, 28, 25

3. 30, 16, 42

4. 24, 70, 74

5. 54, 72, 90

6. 7, 24, 25

7. 12, 16, 30

8. 0.75, 1, 1.25

9. 7.5, 10, 12.5

7.5 SQUARE ROOTS ($\sqrt{\quad}$)

The **square root** of a number is one which when raised to the second power, produces the given number. The square root of 25 is 5 since $5^2 = 25$ and the square roots of 81 is 9 since $9^2 = 81$.

Most numbers do not have exact square roots and we can only get an approximate value for their square roots. These numbers are non-terminating, non-repeating decimals and we call them **Irrational Numbers** as in the examples below.

$$\sqrt{29} = 5.385\ 165 \dots$$

$$\sqrt{73} = 8.544\ 003 \dots$$

$$\pi = 3.141\ 592 \dots$$



A. Use your calculator if necessary to find the square root of each of the following and then state whether each is a rational or irrational number.

1. 345.67

2. 67 432

3. 225

4. 678.432

5. 842.986

6. 3981

7. 731 441

8. 1024

9. 55 225

10. 342 657

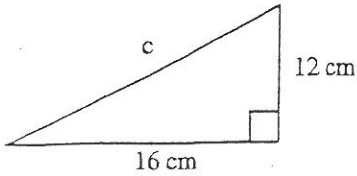
11. 0.001 44

12. 87.609

7.6 CALCULATIONS USING THE THEOREM OF PYTHAGORAS

In algebraic terms the **Theorem of Pythagoras** is written $c^2 = a^2 + b^2$, where 'c' is the hypotenuse of a right angle triangle and 'a' and 'b' are the altitude and base respectively. The examples below show how we can use the Pythagorean Theorem to calculate the third side of a right angle triangle when we are given the lengths of the two other sides.

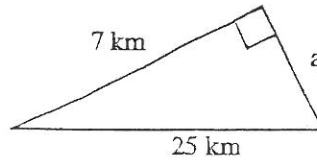
EXAMPLE #1: Calculate the hypotenuse of a right angle triangle where the base is 16 cm and the altitude is 12 cm.



$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 12^2 + 16^2 \\ c^2 &= 144 + 256 \\ \sqrt{c^2} &= \sqrt{400} \leftarrow \text{Find the square root of both sides.} \\ c &= 20 \text{ cm} \end{aligned}$$

∴ the hypotenuse (side 'c') is 20 cm.

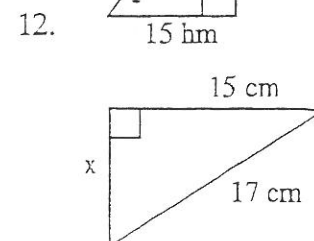
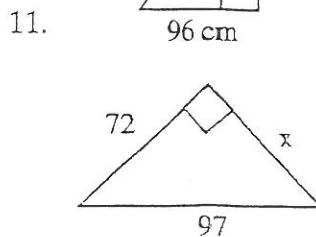
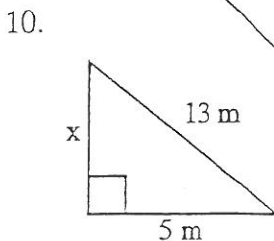
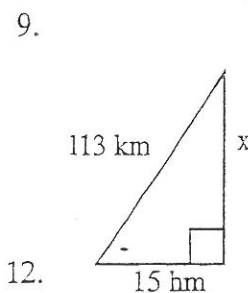
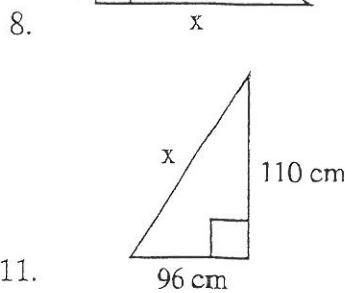
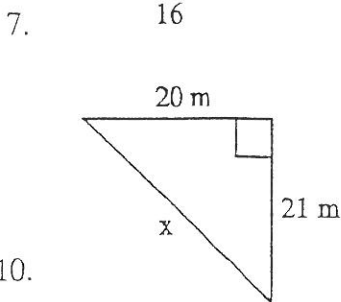
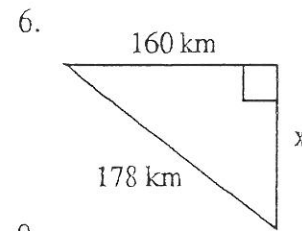
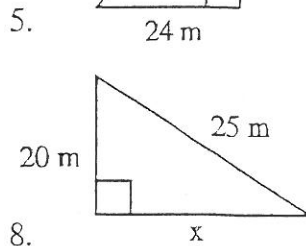
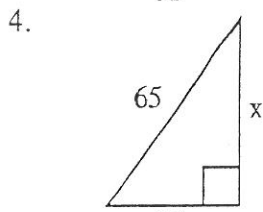
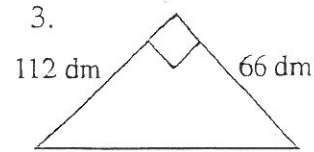
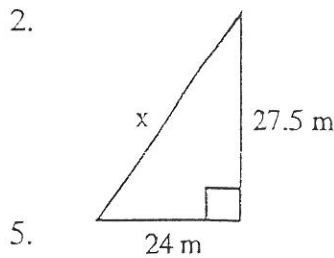
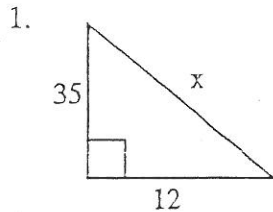
EXAMPLE #2: Find the length of the altitude of a right angle triangle where the base is 7 km and the hypotenuse is 25 km.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 7^2 &= 25^2 \\ a^2 + 49 &= 625 \\ a^2 &= 625 - 49 \\ \sqrt{a^2} &= \sqrt{576} \\ a &= 24 \text{ km} \end{aligned}$$

∴ the altitude is 24 kilometres.

A. Find the length of the missing side labelled 'x' in each right angle triangle below.



B. Draw a labelled diagram to illustrate each problem, then solve for the unknown using the Theorem of Pythagoras.

1. Calculate the length of the diagonal of a rectangle whose sides measure:
 - (a) 14 metres and 48 metres
 - (b) 11 centimetres and 60 centimetres
2. Calculate the length of a rectangle that has the following measurements:
 - (a) diagonal measure 7.5 metres and width measures 4.5 metres
 - (b) diagonal measures 26 centimetres and width measures 10 centimetres

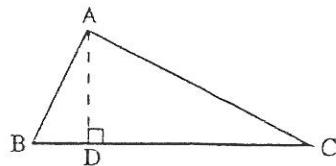
3. A ladder 6.25 metres long rests against a wall at a point 5 metres from the ground. How far is the foot of the ladder from the wall?

4. A square corner lot has sides of 50 metres. Calculate, to the nearest metre, the distance saved in taking a short-cut across the diagonal.

5. Romeo's ladder was 3.125 metres long. When he placed it on the lower edge of Juliette's window, the foot of the ladder was 0.875 metres from the wall. How far was the window above the ground?

6. The diagonal of a rectangular field measure 65 hectometres and the width is 25 hectometres. Find the length of this field.

7. In the diagram below, $AB = 13$ centimetres, $BC = 21$ centimetres and $BD = 5$ centimetres. Using this information, calculate the length of side AC .



8. A pole 7.5 metres high and another pole 4.5 metres high have a wire connecting their tops. If the poles are 4 metres apart, how long is the wire?

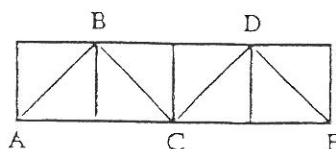
9. State in words the Theorem of Pythagoras.

10. A gate is 12 dm long and 5 dm wide. It has a board to reinforce it that runs along its diagonal. How long is this board?

11. What is the longest straight line that can be drawn on a rectangular piece of paper that is 8 centimetres wide and 15 centimetres long?

12. Calculate the height of an equilateral triangle whose sides are congruent to 10 cm.

13. If each square below represents 1 cm^2 , how long is the zig-zag line from A to E?



7.7 REVIEW OF THE PYTHAGOREAN THEOREM

A. Extra Practice. Solve each of the following.

1. Two telephone poles are 60 metres apart. The poles are 42 metres and 60 metres high. What is the distance between the tops of these two poles?

2. The perimeter of a rectangle is 98 centimetres and one of its sides is 9 centimetres. Find the length of the diagonal of this rectangle.

3. A 12.5 metre ramp extends from the ground to the delivery entrance of a building 3.75 metres above the base of a building. How many metres from the side of the building is the foot of the ramp?

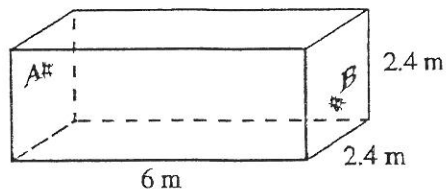
4. A ladder reaches the top of a wall that is 6 metres high. If the base of the ladder is 2.5 metres from the wall, how long is the ladder?

5. A builder is about to cut rafters for a roof of a house. The width of the house is 12 metres and he has to make the roof 2.5 metres higher in the centre than on the sides. If the eaves overhang 0.5 metres, find the total bottom length of each rafter.

6. A baseball diamond is square and the distance between each base is 30 metres. Find the distance from second base to home plate.

7. Can a folded umbrella 98 centimetres long be packed in the bottom of a suitcase 84 centimetres long and 52 centimetres wide?

8. SPIDER'S PROBLEM A room is 6 metres long, 2.4 metres wide and 2.4 metres high. There is a fly on the middle line of one end wall one metre from the floor (at B). There is a spider on the opposite wall one metre from the ceiling and on the middle line of that wall (at A). The fly remains still and the spider catches it by travelling in a straight line from A to B. How far did the spider travel going from A to B?



9. Boat A is 10 km west of boat B. Boat A travels directly north to get to shore. Boat B travels 26 km directly to the same point on shore. How much shorter is Boat A's trip?

10. A square is inscribed in a circle whose diameter is 10 cm. Find the length of the side of the square.

11. A warehouse is 20 m long, 12 m wide and 7 m high. Find (a) the diagonal distance across the floor of the room, and (b) the distance from the southeast corner of the room at the ceiling to the northwest corner of the room at the floor.

12. Find the length of the sides labelled x and y.

