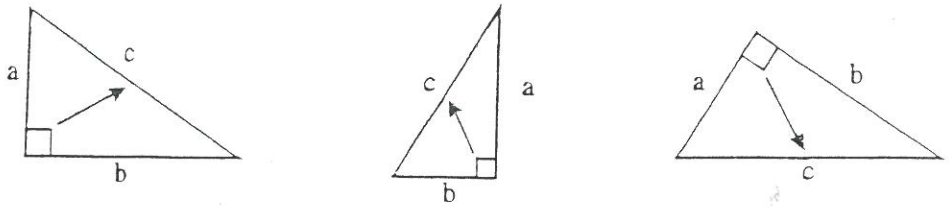
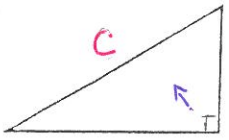
Answer Key

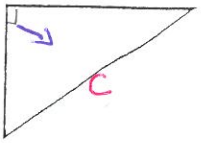
7.2 LABELLING RIGHT ANGLE TRIANGLES

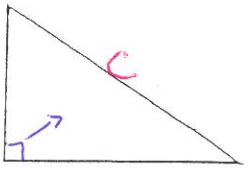
All **right angle triangles** are labelled the same way. The side opposite the right angle is called the **hypotenuse** and is always labelled '*c*'. The other two sides are labelled '*a*' for the **altitude** and '*b*' for the **base** as shown in the three examples below.

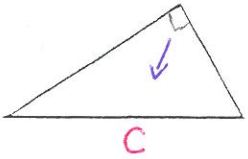


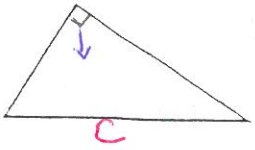
A. Label the hypotenuse of each triangle with the lower case letter '*c*' and each right angle with a capital letter 'C'. (Use a protractor if necessary.)

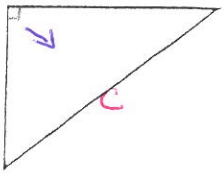
1. 

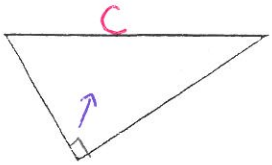
2. 

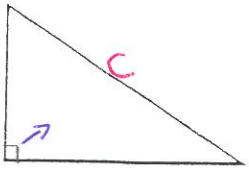
3. 

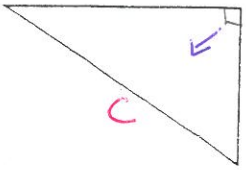
4. 

5. 

6. 

7. 

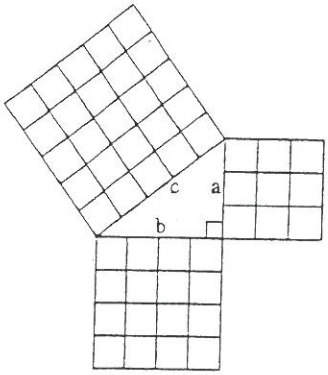
8. 

9. 

7.3 AREA OF SQUARES USING THE PYTHAGOREAN THEOREM

In 552 B.C., in ancient Greece, a man called **Pythagoras** was born and before his death in 507 B.C. he was to become one of the greatest mathematicians of all time. While working with **right-angle triangles**, Pythagoras discovered what we now call the **Pythagorean Theorem.**

In any right-angle triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides. ($c^2 = a^2 + b^2$)



The diagram on the left illustrates that the area of a square drawn on the hypotenuse is equal to the sums of the areas of the squares drawn on the other two sides.

$$c^2 = a^2 + b^2$$
$$5^2 = 3^2 + 4^2$$
$$25 \text{ units}^2 = 9 \text{ units}^2 + 16 \text{ units}^2$$
$$25 \text{ units}^2 = 25 \text{ units}^2$$

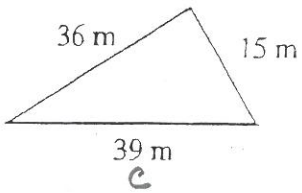
Answer Key

A. Complete the table below to determine whether or not the three numbers given can be the sides of a right angle triangle, or in algebraic terms, is $c^2 = a^2 + b^2$? (The first one is already done for you.)

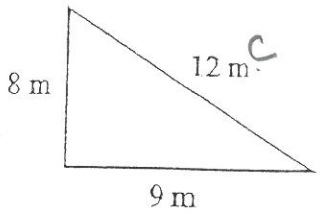
a	a^2	b	b^2	$a^2 + b^2$	c	c^2	Is $c^2 = a^2 + b^2$?
12	144	16	256	400	30	900	NO
5	25	12	144	169	13	169	YES
20	400	21	441	841	29	841	YES
24	576	70	4900	5476	74	5476	YES
7	49	8	64	113	9	81	NO

B. Using the triangles below, complete the table and determine which of the triangles are right angle triangles. (The first one is already done for you.)

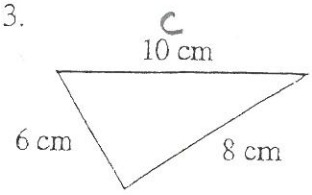
1.



2.



3.



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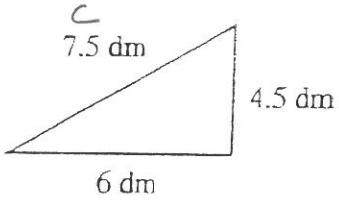
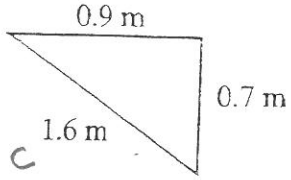
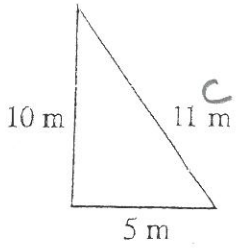


5.



6.



Area of the Square Drawn on One Side	Area of the Square Drawn on Second Side	Sum of the Area of the Squares on 1st Two Sides	Area of the Square Drawn on Longest Side	Is This a Right Angle Triangle? YES or NO?
1. 225 m^2	1296 m^2	1521 m^2	1521 m^2	YES
2. $8^2 = 64$	$9^2 = 81$	$64 + 81 = 145$	$12^2 = 144$	NO
3. $6^2 = 36$	$8^2 = 64$	$36 + 64 = 100$	$10^2 = 100$	YES
4. $4.5^2 = 20.25$	$6^2 = 36$	$20.25 + 36 = 56.25$	$7.5^2 = 56.25$	YES
5. $0.7^2 = 0.49$	$^2 = 0.81$	$0.49 + 0.81 = 1.3$	$1.6^2 = 2.56$	NO
6. $5^2 = 25$	$10^2 = 100$	$25 + 100 = 1.25$	$11^2 = 121$	NO

Answer Key

7.4 PYTHAGOREAN TRIPLETS OR TRIADS

Certain sets of numbers are known as Pythagorean **triplets** or **triads** because they can be used to make up the sides of a right angle triangle. The set of numbers (3, 4, 5) is a triplet or a triad because by using the Theorem of Pythagoras, they satisfy the condition $c^2 = a^2 + b^2$, where the largest number is always the 'c' in our formula or the number associated with the hypotenuse.

EXAMPLE #1: Are the numbers 5, 12 and 13 a Pythagorean triplet?

$$c^2 = a^2 + b^2$$
$$13^2 = 5^2 + 12^2$$
$$169 = 25 + 144$$
$$169 = 169$$

Yes, the numbers 5, 12 and 13 form a triplet.

EXAMPLE #2: Are the numbers 6, 8 and 10 a Pythagorean triad?

$$c^2 = a^2 + b^2$$
$$10^2 = 6^2 + 8^2$$
$$100 = 36 + 64$$
$$100 = 100$$

Yes, the number 6, 8 and 10 form a triad.

A. State whether each of the following is a either a Pythagorean triad or triplet.

1. 5, 6, 7 — $5^2+6^2=7^2$, $25+36=49$, $61 \neq 49$
2. 21, 28, 25 — $21^2+25^2=28^2$, $441+625=784$, $1066 \neq 784$
3. 30, 16, 42 — $30^2+16^2=42^2$, $900+256=1764$, $1156 \neq 1764$
4. 24, 70, 74 — $24^2+70^2=74^2$, $576+4900=5476$, $5476=5476$ yes
5. 54, 72, 90 — $54^2+72^2=90^2$, $2916+5184=8100$, $8100=8100$ yes
6. 7, 24, 25 — $7^2+24^2=25^2$, $49+576=625$, $625=625$ yes
7. 12, 16, 30 — $12^2+16^2=30^2$, $144+256=900$, $400 \neq 900$
8. 0.75, 1, 1.25 — $0.75^2+1^2=1.25^2$, $0.5625+1=1.5625$, $1.5625=1.5625$ yes
9. 7.5, 10, 12.5 — $7.5^2+10^2=12.5^2$, $56.25+100=156.25$, $156.25=156.25$ Yes

7.5 SQUARE ROOTS ($\sqrt{\quad}$)

The **square root** of a number is one which when raised to the second power, produces the given number. The square root of 25 is 5 since $5^2 = 25$ and the square roots of 81 is 9 since $9^2 = 81$.

Most numbers do not have exact square roots and we can only get an approximate value for their square roots. These numbers are non-terminating, non-repeating decimals and we call them **Irrational Numbers** as in the examples below.

$\sqrt{29} = 5.385\ 165\ ...$

$\sqrt{73} = 8.544\ 003\ ...$

$\pi = 3.141\ 592\ ...$

A. Use your calculator if necessary to find the square root of each of the following and then state whether each is a rational or irrational number.

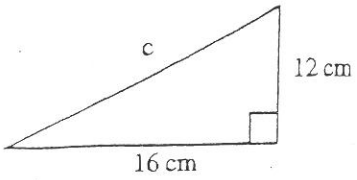
1. 345.67 Q′
2. 67 432 Q′
3. 225 = 15 Q
4. 678.432 Q′
5. 842.986 Q′
6. 3981 Q′
7. 731 441 Q′
8. 1024 = 32 Q
9. 55 225 = 235 Q
10. 342 657 Q′
11. 0.001 44 Q′
12. 87.609 Q′

2/6 Answer Key

7.6 CALCULATIONS USING THE THEOREM OF PYTHAGORAS

In algebraic terms the **Theorem of Pythagoras** is written $c^2 = a^2 + b^2$, where 'c' is the hypotenuse of a right angle triangle and 'a' and 'b' are the altitude and base respectively. The examples below show how we can use the Pythagorean Theorem to calculate the third side of a right angle triangle when we are given the lengths of the two other sides.

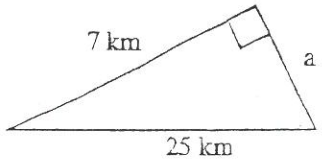
EXAMPLE #1: Calculate the hypotenuse of a right angle triangle where the base is 16 cm and the altitude is 12 cm.



$c^2 = a^2 + b^2$
$c^2 = 12^2 + 16^2$
$c^2 = 144 + 256$
$\sqrt{c^2} = \sqrt{400}$ ←Find the square root of both sides.
$c = 20$ cm

∴ the hypotenuse (side 'c') is 20 cm.

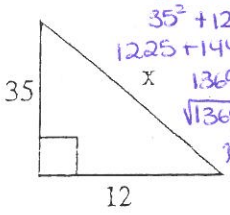
EXAMPLE #2: Find the length of the altitude of a right angle triangle where the base is 7 km and the hypotenuse is 25 km.



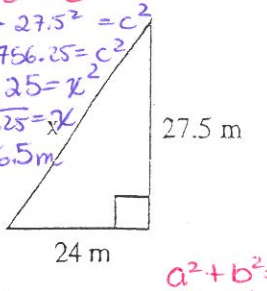
$a^2 + b^2 = c^2$
$a^2 + 7^2 = 25^2$
$a^2 + 49 = 625$
$a^2 = 625 - 49$
$\sqrt{a^2} = \sqrt{576}$
$a = 24$ km

∴ the altitude is 24 kilometres.

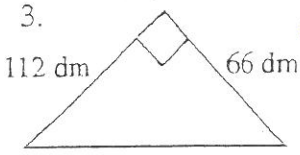
A. Find the length of the missing side labelled 'x' in each right angle triangle below.

1.


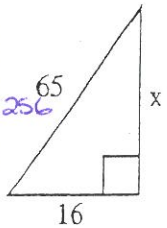
$a^2+b^2=c^2$
$35^2+12^2=x^2$
$1225+144=x^2$
$1369=x^2$
$\sqrt{1369}=x$
$x=37$

2.


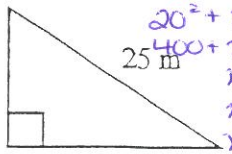
$a^2+b^2=c^2$
$24^2+27.5^2=c^2$
$576+756.25=c^2$
$1332.25=x^2$
$\sqrt{1332.25}=x$
$x=36.5m$

3.


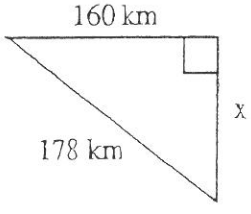
$a^2+b^2=c^2$
$66^2+112^2=x^2$
$4356+12544=x^2$
$16900=x^2$
$\sqrt{16900}=x$
$x=130dm$

4.


$a^2+b^2=c^2$
$16^2+x^2=65^2$
$256+x^2=4225$
$x^2=4225-256$
$x^2=3969$
$x=\sqrt{3969}$
$x=63$

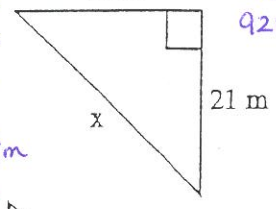
5.


$a^2+b^2=c^2$
$20^2+x^2=25^2$
$400+x^2=625$
$x^2=625-400$
$x^2=225$
$x=\sqrt{225}$
$x=25m$

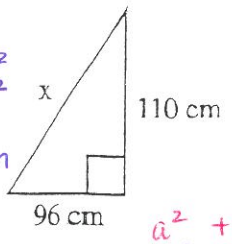
6.


$a^2+b^2=c^2$
$x^2+160^2=178^2$
$x^2+25600=31684$
$x^2=31684-25600$
$x^2=6084$
$x=\sqrt{6084}$
$x=78km$

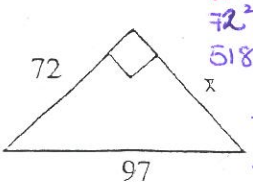
7.
8.
9.

10.


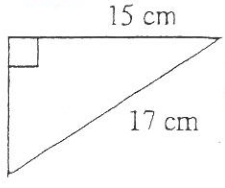
$a^2+b^2=c^2$
$20^2+21^2=x^2$
$400+441=x^2$
$841=x^2$
$\sqrt{841}=x$
$x=29m$

11.


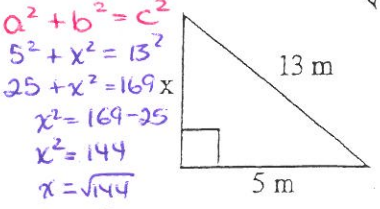
$a^2+b^2=c^2$
$96^2+110^2=x^2$
$9216+12100=x^2$
$21316=x^2$
$\sqrt{21316}=x$
$x=146cm$

12.


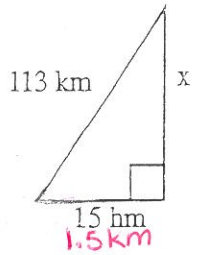
$a^2+b^2=c^2$
$72^2+x^2=97^2$
$5184+x^2=9409$
$x^2=9409-5184$
$x^2=4225$
$x=\sqrt{4225}$
$x=65$

13.


$a^2+b^2=c^2$
$x^2+15^2=17^2$
$x^2+225=289$
$x^2=289-225$
$x^2=64$
$x=\sqrt{64}$
$x=8cm$

14.


$a^2+b^2=c^2$
$5^2+x^2=13^2$
$25+x^2=169$
$x^2=169-25$
$x^2=144$
$x=\sqrt{144}$
$x=12m$

15.


$a^2+b^2=c^2$
$1.5^2+x^2=113^2$
$2.25+x^2=12769$
$x^2=12769-2.25$
$x^2=12766.75$
$\sqrt{x^2}=\sqrt{12766.75}$
$x\approx 112.99$
1.5km