

Questions # 1, 3, 5

The quadratic model

1 Consider the following quadratic function: $f(x) = 2(x - 3)^2 - 2$.

a) Draw the graph of this function.

b) Determine the following properties:

1) the domain

 \mathbb{R}

2) the range

 $[-2, +\infty[$

3) the coordinates of the vertex

 $(3, -2)$

4) the y-intercept

16

5) the extremums

6) the zeros (x-intercepts)

Minimum: -22, 4

7) the interval where the function is increasing

 $[3, +\infty[$

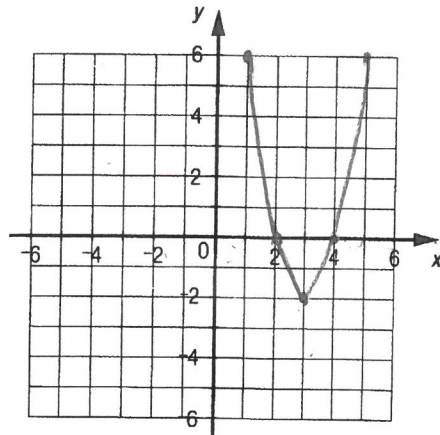
8) the interval where the function is decreasing

 $]-\infty, 3]$

9) the intervals where the function is positive (above x-axis)

 $]-\infty, 2] \cup [4, +\infty[$

10) the interval where the function is negative (below x-axis)

 $[2, 4]$ 

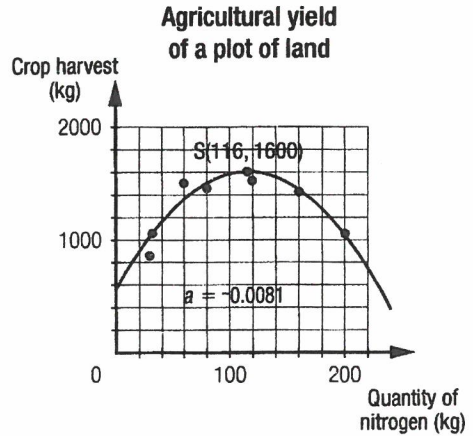
The function $g(x)$ is obtained by a translation of 1 unit toward the left and 3 units upward, followed by a clockwise rotation of 180° centred at the origin.

c) Represent this function in the same Cartesian plane as function f .d) Determine the rule for function g in standard form.

$$\underline{g(x) = -2(x+2)^2 - 1}$$

2 The scatter plot below represents the agricultural yield of crops (in kg) from one hectare of land in terms of the quantity of nitrogen present in the soil (in kg). This relation can be modelled by a quadratic function.

a) Interpret the vertex in this context.



b) Is it true that soil rich in nitrogen produces the best agricultural yield? Justify your answer.

c) A farmer has succeeded in obtaining a more stable yield of crops regardless of the quantity of nitrogen present in the soil. Suggest a rule for a quadratic function that corresponds to this result.

3 a) Write the equation in standard form for each of the curves displayed in the adjacent graph.

$f_1(x) = 5(x+7)^2 - 5$

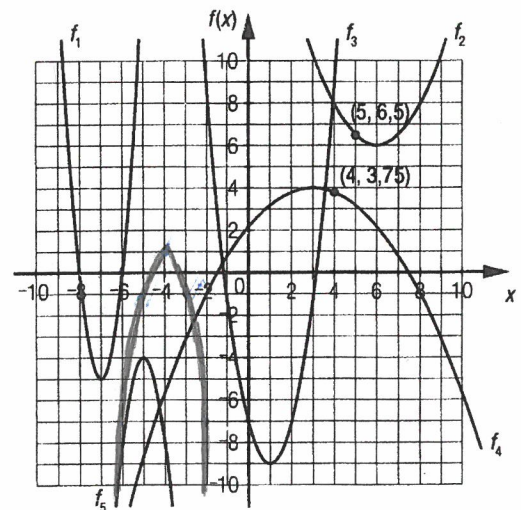
$f_2(x) = 0.5(x-6)^2 + 6$

$f_3(x) = 2(x-1)^2 - 9$

$f_4(x) = -0.25(x-3)^2 + 4$

$f_5(x) = -4(x+5)^2 - 4$

b) In the adjacent Cartesian plane, draw the curve whose equation is $f(x) = -2(x + 4)^2 + 1$.

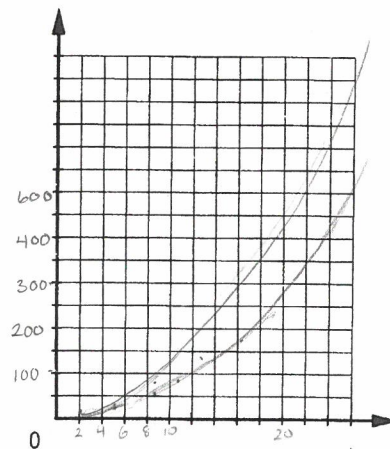


- 4 As part of a security testing operation, the energy being produced by vehicles that collide with a wall at various speeds is being measured. Below are the data collected:

Collision tests

Speed (m/s)	2.8	5.6	8.3	11.1	13.9	16.7	19.4	22.2	25.0
Energy (kJ) for a vehicle of 1500 kg	6	24	51	93	145	209	282	370	469
Energy (kJ) for a vehicle of 2500 kg	10	39	86	154	242	349	470	616	781

Determine the equations for the two curves that best model this situation. Justify your answer.



- 5 Two dolphins are leaping into the air. The second dolphin reaches a height of 0.75 m higher than the first, and in a span of 3 s before the first dolphin. The heights reached (in m) as a function of time (in s) can be modelled by quadratic functions whose parabolas have the same opening.

a) Determine the rule of the function that expresses the height of the jumps of the two dolphins as a function of time. **Vertex (3, 1.5)** Dolphin #2

$$y = a(x-3)^2 + 1.5 \quad (0, -5.7) \quad \rightarrow 0.75 \text{ m higher}$$

$$-5.7 = a(0-3)^2 + 1.5 \quad 1.5 + 0.75 = 2.25$$

$$-5.7 - 1.5 = a(3)^2$$

$$\frac{-7.2}{9} = \frac{9a}{9} \quad \rightarrow \text{Dolphin \#1 } y = -0.8(x-3)^2 + 1.5$$

$$a = -0.8 \quad \text{Dolphin \#2 } y = -0.8(x-1)^2 + 2.25$$

b) What was the initial height of the second dolphin?

$$f(x) = -0.8(x-1)^2 + 2.25$$

$$f(0) = -0.8(0-1)^2 + 2.25$$

$$f(0) = -0.8(1)^2 + 2.25$$

$$f(0) = -0.8 + 2.25$$

$$f(0) = 1.45$$

First dolphin

Time (s)	Height (m)
0	-5.7
1	-1.7
2	0.7
3	1.5
4	0.7
5	-1.7
6	-5.7

$$0 = -0.8(x-3)^2 + 1.5$$

$$-1.5 = -0.8$$

Note:

1st Dolphin has height of 0.75m in 4th sec.
 $x = 4$
 3 seconds earlier makes the vertex of Dolphin #2 at $x = 1$

Exact
 $0.75 = -0.8(x-3)^2 + 1.5$
 $0.75 - 1.5 = -0.8(x-3)^2$
 $-0.75 = \frac{-0.8(x-3)^2}{-0.8}$
 $0.9375 = (x-3)^2$
 $\sqrt{0.9375} = x-3$
 $\sqrt{0.9375} + 3 = x$
 $3 + \sqrt{0.9375}$
 ~ 2.03
 $\uparrow \sim 2 \text{ sec}$
 cannot
 -3 sec in time.