SOLUTIONS: Division Point of a Line Segment
(From Extra Practice 4.2-page R-19)

1) Determine the coordinates of the point dividing line segment $\overline{AB}$ in the given ratio:

a) Workshop $A(1,-4)$ $B(6,4)$

$a = 1$, $b = 3$, $a+b = 4$

$x_1 = -2$, $y_1 = -4$

$\Delta x = x_2 - x_1 = (6) - (-2) = 6 + 2 = 8$

$\Delta y = y_2 - y_1 = (6) - (-2) = 6 + 2 = 8$

$$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x) = (-2) + \left(\frac{1}{4}\right)(8) = (-2) + \left(\frac{8}{4}\right) = (-2) + (2) = 0$$

$$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y) = (-4) + \left(\frac{1}{4}\right)(8) = (-4) + \left(\frac{8}{4}\right) = (-4) + (2) = -2$$

Point $(0,-2)$

b) Workshop $A(2,4)$ $B(12,-1)$

$a = 3$, $b = 2$, $a+b = 5$

$x_1 = 2$, $y_1 = 4$

$\Delta x = x_2 - x_1 = (12) - (2) = 12 - 2 = 10$

$\Delta y = y_2 - y_1 = (-1) - (4) = -1 - 4 = -5$

$$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x) = (2) + \left(\frac{3}{5}\right)(10) = (2) + \left(\frac{30}{5}\right) = (2) + (6) = 8$$

$$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y) = (4) + \left(\frac{3}{5}\right)(-5) = (4) + \left(\frac{-15}{5}\right) = (4) + (-3) = 1$$

Point $(8,1)$

c) Workshop $A(-5,3)$ $B(4,-6)$

$a = 2$, $b = 7$, $a+b = 9$

$x_1 = -5$, $y_1 = 3$

$\Delta x = x_2 - x_1 = (4) - (-5) = 4 + 5 = 9$

$\Delta y = y_2 - y_1 = (-6) - (3) = -6 - 3 = -9$

$$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x) = (-5) + \left(\frac{2}{9}\right)(9) = (-5) + \left(\frac{18}{9}\right) = (-5) + (2) = -3$$

$$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y) = (3) + \left(\frac{2}{9}\right)(-9) = (3) + \left(\frac{-18}{9}\right) = (3) + (-2) = 1$$

Point $(-3,1)$

d) Workshop $A(-12, 17)$ $B(14, 5)$

$a = 7$
$b = 4$
$a + b = 11$

$x_1 = -12$
$y_1 = 17$

$\Delta x = x_2 - x_1$
$= (14) - (-12)$
$= 14 + 12$
$= 26$

$\Delta y = y_2 - y_1$
$= (5) - (17)$
$= 5 - 17$
$= -12$

$$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x)$$
$$= (-12) + \left(\frac{7}{11}\right)(26)$$
$$= (-12) + \left(\frac{7}{11}\right)(26)$$
$$= (-12) + \left(\frac{182}{11}\right)$$
$$= \left(\frac{-132}{11}\right) + \left(\frac{182}{11}\right)$$
$$= \frac{50}{11}$$

$$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y)$$
$$= (17) + \left(\frac{7}{11}\right)(-12)$$
$$= (17) + \left(\frac{7}{11}\right)(-12)$$
$$= (17) + \left(\frac{-84}{11}\right)$$
$$= \left(\frac{187}{11}\right) + \left(\frac{-84}{11}\right)$$
$$= \frac{103}{11}$$

$$Point\ \left(\frac{50}{11}, \frac{103}{11}\right)$$
Or
$$Point\ (4.5, 9.4)$$

2) Given line segment $\overline{MN}$:

a) Point $(1, -1)$ is the midpoint of $\overline{MN}$. If the coordinates of M are $(-1, 3)$, determine those of N.

$$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$$
$$\left((1) = \frac{(-1) + (x)}{2},\ (-1) = \frac{(3) + (y)}{2}\right)$$
$$\left((2)(1) = \left(\frac{(-1) + (x)}{2}\right)(2),\ (2)(-1) = \left(\frac{(3) + (y)}{2}\right)(2)\right)$$
$$(2 = -1 + x,\ -2 = 3 + y)$$
$$(3 = x,\ -5 = y)$$

$$Midpoint\ (3, -5)$$

b) Point $(5, 1)$ is the midpoint of $\overline{MN}$. If the coordinates of N are $(1, -3)$, determine those of M.

$$\left(x_m = \frac{x_1 + x_2}{2},\ y_m = \frac{y_1 + y_2}{2}\right)$$
$$\left((5) = \frac{(1) + (x)}{2},\ (1) = \frac{(-3) + (y)}{2}\right)$$
$$\left((2)(5) = \left(\frac{(1) + (x)}{2}\right)(2),\ (2)(1) = \left(\frac{(-3) + (y)}{2}\right)(2)\right)$$
$$(10 = 1 + x,\ 2 = -3 + y)$$
$$(11 = x,\ 5 = y)$$

$$Midpoint\ (11, 5)$$

3) Point $C(-8,3)$ is the centre of a circle . A diameter if this circle has A and B as its endpoints. The coordinates of point A are $(3,5)$. Find the coordinates of point B.

$$\left(x_m = \tfrac{x_1+x_2}{2}\ ,\ y_m = \tfrac{y_1+y_2}{2}\right)$$
$$\left((-8) = \tfrac{(3)+(x)}{2}\ ,\ (3) = \tfrac{(5)+(y)}{2}\right)$$
$$\left((2)(-8) = \left(\tfrac{(3)+(x)}{2}\right)(2)\ ,\ (2)(3) = \left(\tfrac{(5)+(y)}{2}\right)(2)\right)$$
$$(-16 = 3+x\ ,\ 6 = 5+y)$$
$$(-19 = x\ ,\ 1 = y)$$

Midpoint $(-19,1)$

4) Find the coordinates of the points that divide the line segment whose endpoints are $A(-8,10)$ and $B(10,-2)$ into three congruent sections.

The First Division Point $A(-8,10)$, $B(10,-2)$, $\frac{1}{3}$

Workshop $A(-8,\ 10)$ $B(10,-2)$

$a = 1$
$b = 2$
$a+b = 3$

$x_1 = -8$
$y_1 = 10$

$\Delta x = x_2 - x_1$
$= (10) - (-8)$
$= 10 + 8$
$= 18$

$\Delta y = y_2 - y_1$
$= (-2) - (10)$
$= -2 - 10$
$= -12$

$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x)$
$= (-8) + \left(\frac{1}{3}\right)(18)$
$= (-8) + \left(\frac{1}{3}\right)(18)$
$= (-8) + \left(\frac{18}{3}\right)$
$= (-8) + (6)$
$= -2$

$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y)$
$= (10) + \left(\frac{1}{3}\right)(-12)$
$= (10) + \left(\frac{1}{3}\right)(-12)$
$= (10) + \left(\frac{-12}{3}\right)$
$= (10) + (-4)$
$= 6$

First Division Point $(-2,6)$

The Second Division Point $A(-8,10)$, $B(10,-2)$, $\frac{2}{3}$

Workshop $A(-8,\ 10)$ $B(10,-2)$

$a = 2$
$b = 1$
$a+b = 3$

$x_1 = -8$
$y_1 = 10$

$\Delta x = x_2 - x_1$
$= (10) - (-8)$
$= 10 + 8$
$= 18$

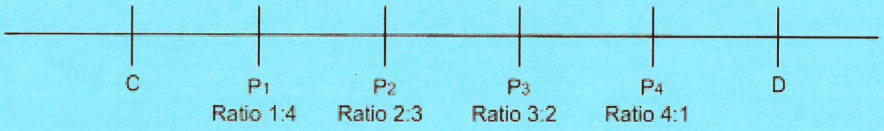
$\Delta y = y_2 - y_1$
$= (-2) - (10)$
$= -2 - 10$
$= -12$

$x_D = x_1 + \left(\frac{a}{a+b}\right)(\Delta x)$
$= (-8) + \left(\frac{2}{3}\right)(18)$
$= (-8) + \left(\frac{2}{3}\right)(18)$
$= (-8) + \left(\frac{36}{3}\right)$
$= (-8) + (12)$
$= 4$

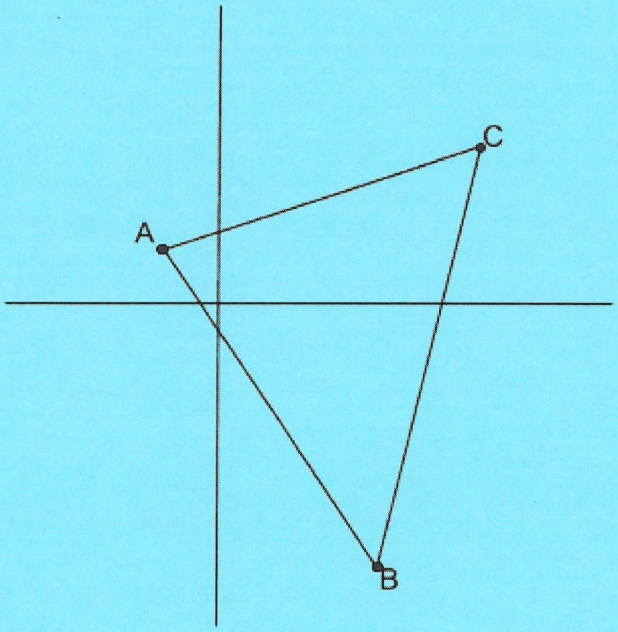
$y_D = y_1 + \left(\frac{a}{a+b}\right)(\Delta y)$
$= (10) + \left(\frac{2}{3}\right)(-12)$
$= (10) + \left(\frac{2}{3}\right)(-12)$
$= (10) + \left(\frac{-24}{3}\right)$
$= (10) + (-8)$
$= 2$

Second Division Point $(4,2)$

5) A line segment CD is divided into five congruent points P_1, P_2, P_3, P_4. In what ratio does each of the points divide line segment CD.



6) $A(-1,1)$, $B(3,-5)$, $C(5,3)$, are the vertices of a triangle. Calculate the length of each of the medians of the triangle.



Midpoint of $\overline{AB}$

$$\left(x_m = \frac{x_1+x_2}{2},\ y_m = \frac{y_1+y_2}{2}\right)$$
$$\left(x_m = \frac{(-1)+(3)}{2},\ y_m = \frac{(1)+(-5)}{2}\right)$$
$$\left(x_m = \frac{2}{2},\ y_m = \frac{-4}{2}\right)$$
$$(x_m = 1,\ y_m = -2)$$
$$(1, -2)$$

Midpoint of $\overline{AC}$

$$\left(x_m = \frac{x_1+x_2}{2},\ y_m = \frac{y_1+y_2}{2}\right)$$
$$\left(x_m = \frac{(-1)+(5)}{2},\ y_m = \frac{(1)+(3)}{2}\right)$$
$$\left(x_m = \frac{4}{2},\ y_m = \frac{4}{2}\right)$$
$$(x_m = 2,\ y_m = 2)$$
$$(2, 2)$$

Midpoint of $\overline{BC}$

$$\left(x_m = \frac{x_1+x_2}{2},\ y_m = \frac{y_1+y_2}{2}\right)$$
$$\left(x_m = \frac{(3)+(5)}{2},\ y_m = \frac{(-5)+(3)}{2}\right)$$
$$\left(x_m = \frac{8}{2},\ y_m = \frac{-2}{2}\right)$$
$$(x_m = 4,\ y_m = -1)$$
$$(4, -1)$$